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BOUNDING MULTIPLICITY BY SHIFTS IN THE TAYLOR RESOLUTION

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ABSTRACT. A weaker form of the multiplicity conjecture of Herzog, Huneke, and Srinivasan is proven for two classes of monomial ideals: quadratic monomial ideals and squarefree monomial ideals with sufficiently many variables relative to the Krull dimension. It is also shown that tensor products, as well as Stanley-Reisner ideals of certain unions, satisfy the multiplicity conjecture if all the components do. Conditions under which the bounds are achieved are also studied.

1. Introduction. In this paper we examine a relaxation of the multiplicity conjecture by using non-minimal free resolutions.

Throughout the paper we work with the polynomial ring $S = \mathbf{k}[x_1, \dots, x_n]$ over an arbitrary field $\mathbf{k}$. If $I \subset S$ is a homogeneous ideal, then the ($\mathbb{Z}$-graded) *Betti numbers* of S/I, $\beta_{i,j} = \beta_{i,j}(S/I)$, are the invariants that appear in the minimal free resolution of S/I as an S-module:

$$0 \to \bigoplus_j S(-j)^{\beta_{l,j}} \to \cdots \to \bigoplus_j S(-j)^{\beta_{2,j}} \to \bigoplus_j S(-j)^{\beta_{1,j}} \to S \to S/I \to 0.$$

Here $S(-j)$ denotes S with grading shifted by j and l denotes the length of the resolution. In particular, $l \geq \operatorname{codim}(I)$.

Our main objects of study are the *maximal and minimal shifts* in the resolution of S/I defined by $M_i = M_i(S/I) = \max\{j : \beta_{i,j} \neq 0\}$ and $m_i = m_i(S/I) = \min\{j : \beta_{i,j} \neq 0\}$ for $i = 1, \dots, l$, respectively. The following conjecture due to Herzog, Huneke, and Srinivasan [**6**] is known as the multiplicity conjecture.

Conjecture 1.1. *Let $I \subset S$ be a homogeneous ideal of codimension c. Then the multiplicity of S/I, $e(S/I)$, satisfies the following upper bound:*

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