

## COMPUTING GORENSTEIN COLENGTH

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**ABSTRACT.** Given an Artinian local ring  $R$ , we define (in [1]) its Gorenstein colength  $g(R)$  to measure how closely we can approximate  $R$  by a Gorenstein Artin local ring. In this paper, we show that  $R = T/\mathfrak{b}$  satisfies the inequality  $g(R) \leq \lambda(R/\text{soc}(R))$  in the following two cases: (a)  $T$  is a power series ring over a field of characteristic zero and  $\mathfrak{b}$  an ideal that is the power of a system of parameters or (b)  $T$  is a 2-dimensional regular local ring with infinite residue field and  $\mathfrak{b}$  is primary to the maximal ideal of  $T$ .

In the first case, we compute  $g(R)$  by constructing a Gorenstein Artin local ring mapping onto  $R$ . We further use this construction to show that an ideal that is the  $n$ th power of a system of parameters is directly linked to the  $(n-1)$ st power via Gorenstein ideals. A similar method shows that such ideals are also directly linked to themselves via Gorenstein ideals.

**1. Introduction.** Let us first recall the definition of Gorenstein colength and review some of its basic properties from [1] in this section.

**Definition 1.1.** Let  $(R, \mathfrak{m}, \mathfrak{k})$  be an Artinian local ring. Define the Gorenstein colength of  $R$ , denoted  $g(R)$  as:  $g(R) = \min\{\lambda(S) - \lambda(R) : S \text{ is a Gorenstein Artin local ring mapping onto } R\}$ , where  $\lambda(-)$  denotes length.

The main questions one would like to answer are the following:

**Question 1.2.**

- a) How does one intrinsically compute  $g(R)$ ?
- b) How does one construct a Gorenstein Artin local ring  $S$  mapping onto  $R$  such that  $\lambda(S) - \lambda(R) = g(R)$ ?

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