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THE AUSLANDER-BRIDGER FORMULA AND THE GORENSTEIN PROPERTY FOR COHERENT RINGS

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ABSTRACT. The concept of Gorenstein dimension, defined by Auslander and Bridger for finitely generated modules over a Noetherian ring, is studied in the context of finitely presented modules over a coherent ring. A generalization of the Auslander-Bridger formula is established and is used as a cornerstone in the development of a theory of coherent Gorenstein rings.

1. Introduction. In addressing a problem posed by Glaz ([**11, 12**]), Hamilton and the second author give a definition of Cohen-Macaulay for commutative rings which agrees with the usual notion for Noetherian rings with the property that every coherent regular ring is Cohen-Macaulay [**13**]. (A quasi-local ring is defined to be *regular* if every finitely generated ideal has finite projective dimension.) A natural question is whether there is a reasonable concept of Gorenstein for commutative rings such that every coherent regular ring is Gorenstein and every coherent Gorenstein ring is Cohen-Macaulay. In this paper, we develop such a theory of coherent Gorenstein rings which mirrors much of the theory in the Noetherian case. Central to this development is the concept of Gorenstein dimension (G-dimension, for short), first introduced in the context of finitely generated modules over Noetherian rings by Auslander and Bridger [**1**]. In particular, we prove the following generalization of the Auslander-Bridger formula for coherent rings using a notion of depth for arbitrary quasi-local rings developed by Barger [**2**], Hochster [**14**], and Northcott [**18**]:

Theorem 1.1. *Let R be a quasi-local coherent ring and M a finitely presented R-module of finite G-dimension. Then*

$$\operatorname{depth} M + \operatorname{Gdim}_R M = \operatorname{depth} R.$$

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