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ARTINIANNESS OF LOCAL COHOMOLOGY

M. HELLUS AND J. STÜCKRAD

ABSTRACT. Let $R = k[[x, y, u, v]]$ over a field k, $I = \langle u, v\rangle$ and $p = xu + yv$. Hartshorne has proved that $H^2_I(R/pR)$ is not artinian. We show that the same is true for *every* element p of $(x, y)R$. In fact, we show an even stronger statement. We use Matlis duals of local cohomology modules.

1. Introduction. It is an interesting question to determine if a given local cohomology module $H^i_I(M)$ is artinian, where I is an ideal of a local ring (R, m) and M is a finite R-module; this is one of Huneke's problems on local cohomology (see [**3**, third problem]). In this note, we prove that a large class of local cohomology modules is not artinian:

Theorem 1.1. *Let (R, m) be a local, complete ring, $n = \dim(R) \geq 4$. Let I be an ideal of R of height $n-2$ such that $H^{n-1}_I(R) = H^n_I(R) = 0$. Let $a, b \in R$ such that $(a, b)R$ is a prime ideal of height two and such that a, b defines a system of parameters for R/I. Then, for every $p \in (a, b)R$, $H^{n-2}_I(R/pR)$ is not artinian.*

We actually prove something stronger: the Matlis dual $D(H^{n-2}_I(R/pR))$ has infinitely many associated prime ideals and is therefore not noetherian.

Theorem 1.1 immediately specializes to the following result which was proved by Hartshorne ([**2**, Section 3]): $H^2_I(R)$ is not artinian, where $R = k[[x, y, u, v]]/(xu+yv)$, k is a field and $I \subseteq R$ is the ideal generated by the classes of u and v in R; in fact, according to Theorem 1.1, we can replace $xu+yv$ by any element of $(x, y)R$ and the statement is still true.

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