

BOUNDS FOR THE CASTELNUOVO-MUMFORD REGULARITY

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ABSTRACT. We extend the “linearly exponential” bound for the Castelnuovo-Mumford regularity of a graded ideal in a polynomial ring $K[x_1, \dots, x_r]$ over a field (established by Galligo and Giusti in characteristic 0 and recently, by Caviglia-Sbarra for arbitrary K) to graded submodules of a graded module over a homogeneous Cohen-Macaulay ring $R = \bigoplus_{n \geq 0} R_n$ with artinian local base ring R_0 . As an application we get a “linearly exponential” bound for the Castelnuovo-Mumford regularity of a graded R -module M in terms of the degrees which occur in a minimal free presentation of M .

1. Introduction. The first result on Castelnuovo-Mumford regularity, proved a long time before this notion even was created, is a bounding result: Castelnuovo’s “bound on the regularity of the vanishing ideal of a projective space curve” (cf [6]).

Similarly, the classical controversy around the “problem of the finitely many steps” (cf [12, 13]) which grew out of Hilbert’s “Syzygientheorie”, also may be understood as the question for a regularity bound: Do the degrees which occur in a minimal free presentation of a (finitely generated) graded module (over a polynomial ring over a field) bound the Castelnuovo-Mumford regularity of this module (cf 6.6 for more details)?

When Mumford introduced the notion of “Castelnuovo regularity” (cf [16]) he first proved a bounding result which is of basic significance for the construction of Hilbert- and Picard schemes.

Since then, the search for regularity bounds has become a theme of constant interest, motivated by the crucial rôle played by these

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