JOURNAL OF COMMUTATIVE ALGEBRA
Volume 1, Number 1, Spring 2009

CELLULAR RESOLUTIONS OF COHEN-MACAULAY MONOMIAL IDEALS

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ABSTRACT. We investigate monomial labelings on cell complexes, giving a minimal cellular resolution of the ideal generated by these monomials, and such that the associated quotient ring is Cohen-Macaulay. We introduce a notion of such a labeling being maximal. There is only a finite number of maximal such labelings for each cell complex, and we classify these for trees, subdivisions of polygons, and some classes of selfdual polytopes.

1. Introduction. In this paper we study cellular resolutions of monomial ideals which have a Cohen-Macaulay quotient ring. Cellular resolutions of monomial ideals, introduced in [2] and [3], is a very natural technique for constructing resolutions of monomial ideals, and appealing in its blending of topological constructions, combinatorics and algebraic ideas. Much activity has centered around it in the last decade, and good introductions and surveys may be found in [8] and [11]. Usually one starts with a monomial ideal and finds a suitable labeled cell complex giving a (preferably minimal) resolution of the monomial ideal. It was hoped that a minimal resolution of a monomial ideal was always cellular, but this was shown recently not to be so, [10].

Here we turn this around and start with the cell complex, and ask what monomial labellings are such that this cell complex gives a minimal cellular resolution of the ideal formed by the monomials in the labeling. To limit the task we assume that the monomial labeling is such that the monomial quotient ring is Cohen-Macaulay, and the cell complex gives a minimal cellular resolution of it. Such a labeling will be called a Cohen-Macaulay (CM) monomial labelling.

For a given cell complex, we define a notion of maximal CM monomial labeling. These are essentially labellings by monomials $\mathbf{x}^{\mathbf{a}_i}$ in a

2000 AMS *Mathematics subject classification.* Primary Primary 13D02. Secondary 13F55, 05E99.

Received by the editors on December 19, 2007, and in revised form on April 2, 2008.

DOI:10.1216/JCA-2009-1-1-57