

OPEN PROBLEMS ON SYZYGIES AND HILBERT FUNCTIONS

IRENA PEEVA AND MIKE STILLMAN

1. Introduction.

In this paper we list a number of open problems and conjectures on Hilbert functions and syzygies. Some of the problems are closely related to Algebraic Geometry, Combinatorics, and Hyperplane Arrangements Theory.

Our aim is to stimulate interest, rather than to give a complete survey. When describing a problem, we sometimes state one or two related results, and give pointers to a few references, rather than giving an exhaustive list of references and what is known. A detailed survey of the covered topics would make the paper far longer than we (and perhaps, the readers) could handle.

Our list of problems is certainly not complete. We have focused on problems that we see as most exciting, or important, or popular. We present three types of problems: Conjectures, Problems, and Open-Ended Problems. Some of the problems and especially the Open-Ended problems are general problems which point to interesting directions for exploration.

The books [35] and [95] contain expository papers on some of the problems and related topics. Section 17 is a (probably non-complete) list of helpful books. [3, 38, 63, 71, 72, 80, 116, 117] provide lecture notes. A good way to get a feel of the recent research is to browse the web pages of the mathematicians working in this area.

2. Notation.

Throughout k stands for a field. For simplicity, we assume that k is algebraically closed and has characteristic 0. However, many of the open problems and conjectures make sense without these assumptions. In the paper, the polynomial ring $S = k[x_1, \dots, x_n]$ is graded by

Received by the editors on May 16, 2008, and in revised form on August 1, 2008.
DOI:10.1216/JCA-2009-1-1-159 Copyright ©2008 Rocky Mountain Mathematics Consortium