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On a Theorem of Lueroth

By

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Let K be a field of degree of transcendency 1 over a field k, then the well-known theorem of Lüroth[1] asserts that K is a simple extension of k, when K is contained in such a field. Now we shall present three different proofs for a generalization of this theorem which are connected closely by the general theory of *Picard varieties*[2]. The present author interests more in the different methods of proof rather than the result itself, which can be stated as follows:

Let K be a field of degree of transcendency 1 *over a field k, then K is a simple extension of k, whenever K is contained in a purely transcendental extension of k.*

We assume thereby that k is a perfect field in order to assure the existence of a non-singular model for K over k; although the theorem is true for an arbitary field k, as we can see from another aspect.

Now let $(t)=(t_1,\dots, t_m)$ be a set of independent variables over k, then since K is an intermediary field of $k(t)$ and k, it can be generated over k by a finite set of quantities. Since we have assumed k as a perfect field, there exists a complete non-singular Curve $\boldsymbol{C}$ with a generic Point $\boldsymbol{P}$ over k such that

$$K=k(\boldsymbol{P}).$$

I was asked in a certain occasion to generalize Lüroth's theorem from Prof. Akizuki; and the publication of this note has been advised also by him. In this note we shall stick in results and terminologies to Weil's book: Foundations of algebraic geometry, Am. Math. Soc. Colloq., vol. 29 (1946).

1) Beweis eines Satzes über rationale Curven, Math. Ann. 9 (1876). See also B. L. v. d. Waerden, Moderne Algebra, § 63.

2) The first two proofs A and B concern clearly with this theory; the same is true for the proof C. See my papers, On the Picard varieties attached to algebraic varieties, to appear in the Amer. J. of Math.; Algebraic correspondences between algebraic varieties, to appear in the Jap. J. ot Math.