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Note on the boundedness of solutions of a system of differential equations

By

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In the foregoing papers[1] we have discussed the existence of a periodic solution of the non-linear differential equation and we have obtained a sufficient condition[2] for the *boundedness* of solutions in order to use Massera's theorem.[3] That condition is analogous to Okamura's theorem[4] for the possibility of the continuation of solutions. In this paper we will obtain necessary and sufficient conditions for the boundedness of solutions of such a type that we have discussed formerly on the evaluation of the derivatives of solutions of a differential equation of the second order.[5]

Now we consider a system of differential equations,

$$\frac{dy_i}{dx}=f_i(x, y_1, \cdots, y_n) \quad (i=1, 2, \cdots, n), \tag{1}$$

where $f_i(x, y_1, \cdots, y_n)$ are *continuous* in the domain

$$\Delta_1: \; 0\leqq x<\infty, \qquad -\infty<y_i<+\infty \, (i=1, 2, \cdots, n).$$

And we discuss the boundedness for $0\leqq x<\infty$ of solutions starting from $x=0$.

To simplify our statements, here we give at first a list of the domains and their symbols used in the following.

(1) Yoshizawa; "*On the non-linear differential equation*", These Memoirs, Vol. 28 (1953), pp. 133-141 and "*Note on the existence theorem of a periodic solution of the non-linear differential equation*", *ibid.*, pp. 153-159.

(2) Yoshizawa; *ibid.*, p.153.

(3) Massera; "*The existence of periodic solutions of systems of differential equations*", Duke Math. Journal, Vol. 17 (1950), pp. 457-475.

(4) Okamura; Functional Equations (in Japanese), Vol. 32 (1942) or Yoshizawa; *ibid.*, p. 139.

(5) Yoshizawa; "*On the evaluation of the derivatives of solutions of $y''=f(x, y, y')$*", These Memoirs, Vol. 28 (1953), pp.27-32.