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On a two-dimensional projectively connected space in the wide sense with torsion

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§ **1.** Consider a two-dimensional space R_2 whose moving point is designed by parameters u and v. Suppose that a curve C drawn on R_2 is developed on a curve Γ of a three-dimensional projective space S_3 by means of the projective connexion $\Gamma^{\beta}_{\alpha r}$ (α, $\beta=0, 1, 2, 3$; $r=1, 2$), the moving frame of reference $[A\, A_1 A_2 A_3]$ along to Γ being defined by

$$(1\cdot1) \qquad dA_\alpha=\Gamma^{\beta}_{\alpha r} du^r A_\beta\,,$$
$$\begin{pmatrix} A_0\equiv A\,; & u^1=u, \quad u^2=v \\ \alpha,\ \beta=0,\ 1,\ 2,\ 3\,; & r=1,\ 2 \end{pmatrix}.$$

We shall denote hereafter by α, β, $\cdots$ the suffixes which take the values 0, 1, 2, 3; by $a, b,\cdots i, j,\cdots r, s,\cdots$ those which take the values 1, 2, except for § 2.

We use the following notations:

$$(1\cdot2) \qquad H_{rs}=\frac{1}{2}(\Gamma^{3}_{rs}+\Gamma^{3}_{sr}),$$

$$(1\cdot3) \qquad H=det\,|H_{rs}|\,,$$

$$(1\cdot4) \qquad H_{rst}=\frac{1}{3}(\varDelta_r H_{st}+\varDelta_s H_{tr}+\varDelta_t H_{rs}),$$

where
$$\varDelta_t H_{rs}=\frac{\partial H_{rs}}{\partial u^t}-\Gamma^{a}_{rt}H_{as}-\Gamma^{a}_{st}H_{ra}\,,$$

$$(1\cdot5) \qquad H_r=\frac{1}{4}H_{rst}H^{st},$$

where H^{st} is the normalized cofactor of the element H_{ts} in H,

$$(1\cdot6) \qquad K_{rst}=H_{rst}-H_r H_{st}-H_s H_{tr}-H_t H_{rs}\,.$$

Let $R^{\beta}_{\alpha rs}$ be the curvature tensor relative to the connexion $\Gamma^{\beta}_{\alpha r}$, that is