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Some remark on rational points

By

Hajime NISHIMURA

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§ **1.** It seems to us that, in spite of their importance, little is known about properties of rational points on algebraic varieties. In this short note we shall prove[1)]

THEOREM. *Let* U^r, V *be abstract varieties,* V *be complete,* π *be a rational function defined on* U *with values in* V, *and* k *be a common field of definition for* U, V *and* π. *If* U *has a rational point* P *over* k *which is simple on* U, *then* V *has also a rational point over* k.

We begin by two lemmas which are proved in elementary way.

LEMMA 1. *Let* U^r *be an algebraic variety in* S^n, $P=(x)$ *be a simple point of* U *and* k *be a field over which* U *and* (x) *are rational. Then there is a subvariety* W *of dimension* $r-1$ *with the following properties:*

(i) *P is contained in W as a simple point,*

(ii) *W is defined over a purely transcendental extention* $k(u)$ *of* k.

(iii) *every rational function* π *defined on* U *is regular along* W.

PROOF. Let H be the hyperplane of S^n defined by the equation

$$\sum_{i=1}^{n} u_i(X_i - x_i) = 0,$$

where (u) is a set of independent variables over k.

Then, as P is simple on U and H is transversal to U at P, it

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1) This problem is proposed to us by Y. Nakai.

2) It is noted that, instead of using a component of hyperplane section of U, we may take as W the most general hypersurface section of sufficiently high degree containing P, which is itself absolutely irreducible subvariety. See M. Nishi and Y. Nakai, "On the hypersurface sections of algebraic varieties embedded in a projective space." Mem. Coll. Sci. Univ. of Kyoto, vol. XXIX, 1955.