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On the imbedding problem of abstract varieties in projective varieties

By

Masayoshi NAGATA

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Since the notion of abstract varieties was introduced by Weil [3], it was asked whether every abstract variety can be imbedded (biregularly) in a projective variety or not. Though the writer hoped to solve the question affirmatively, he found unfortunately a counter example against the question. Indeed, there exists a non-singular abstract variety V (which is not complete) which has two different points P and P' such that if a function f on V is well defined at both P and P', then $f(P)=f(P')$.

After some preliminaries in § 1, we shall show the example in § 2. In § 3, we shall give a condition for an abstract variety to be imbedded in a projective variety and then we shall show in § 4 that even when a monoidal transform of a non-singular abstract variety V can be imbedded in a projective variety, V may not be imbedded in any projective variety. By the way, we shall give some remarks in § 5.

Terminology. Since the notion of abstract varieties corresponds to the notion of models in the sense of Nagata [1], we shall explain in terminology on models as in Nagata [1].

Results assumed to be known: Besides some basic results on rings and models, we shall make use of the criterion of simplicity by Jacobian matrix. Further, in §§ 4–5, we shall make use of some basic results on monoidal transformations and quadratic transformations (see [1, IV]).

§ 1. Some preliminary lemmas

LEMMA 1. *Let M and M' be models of the same function field. Then $M \cup M'$ is again a model if and only if the join $J(M, M')$ of M and M' is contained in $M \cap M'$.*