J. Math. Kyoto Univ.
1-3 (1962) 403-409.

On a theorem of Weitzenböck in invariant theory

By

C. S. Seshadri

(Communicated by Prof. M. Nagata, March 14, 1962)

Let V be an affine variety and G an algebraic group acting on V (on the left). Let R be the coordinate ring of V and $I_G(R)$ its subring of G-invariants. If the characteristic of the ground field is zero, and G is semi-simple, it is now a well-known result that $I_G(R)$ is finitely generated over the ground field (cf. [2] or [4]). The same result is also known to be true without any hypothesis on the characteristic of the ground field if G is a torus group (cf. [2]). In these cases, the essential part of the proof is that there is a canonical projection operator of the space of regular functions on the group G onto the space of constants, which can be extended to a projection operator of R onto $I_G(R)$ and then using the fact that the ring of polynomials in a finite number of variables over a field is Noetherian, we get the result. There is a less known result of Weitzenböck (cf. [3]), which says that if V is the affine space and G an algebraic 1-parameter group acting on V by linear transformations, $I_G(R)$ is again finitely generated, provided that the ground field is of characteristic zero; and it is realized easily that this result is equivalent to the assertion that $I_G(R)$ is finitely generated for the particular case $G=$the additive group G_a. This result is no longer true if V is an arbitrary affine variety (this is a consequence of the famous counter example of Nagata (cf. [1] or [2]) showing that $I_G(R)$ need not be, in general, finitely generated).

In this note, we prove a result (cf. Theorem 1) which is valid