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On a compactification of Green spaces. Dirichlet problem and theorems of Riesz type

By

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Introduction

In this paper motivated by a recent paper of S. Mori [9] we shall study a compactification of Green spaces under the use of L. Naim's results [10] on Martin spaces and discuss the Dirichlet problem and some applications to the functiontheory.

We consider, as the basic space, a Green space R and define in §2 a compactification R^* of R, maximal ideal space of a normed ring $\mathfrak{S}$. Every non-negative continuous superharmonic function on R can be extended continuously onto R^* (Lemma 3). The ideal boundary R^*-R has a compact subset with remarkable properties, which is called, after H. L. Royden, the harmonic boundary (sec. 4).

In §3 we treat Dirichlet problems for functions given on the harmonic boundary. Since it is shown that R^* is not metrisable (Theorem 2), the usual Perron's approach must be somewhat modified, in particular for the discussion of solvability, but the results are quite similar.

As applications we show finally in §4 a theorem (Theorem 11) of Riesz type for several complex variables and refer to a Constantinescu-Cornea's theorem [4] on open Riemann surfaces which, in case of the unit circle, reduces exactly to the theorem of Riesz-Lusin-Privaloff-Frostman-Nevanlinna.