J. Math. Kyoto Univ.
1-3 (1962) 327-355.

Analyticity of the fundamental solutions of hyperbolic systems*

By

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(Received Feb. 18, 1962)

1. Introduction

Consider a hyperbolic system with analytic coefficients

$$M[u]=\frac{\partial u}{\partial t}-\sum_{k=1}^{n}A_k(x,t)\frac{\partial u}{\partial x_k}-B(x,t)u=0\,, \tag{1.1}$$

where the word hyperbolic means the following: $\sum A_k(x, t)\xi_k$ has, for any real $\xi\neq 0$ and all (x, t), N distinct real eigenvalues, $\lambda_1(x, t; \xi)$, $\cdots$, $\lambda_N(x, t; \xi)$. Here u are vector-valued functions with N components; A_k and B are real analytic functions of $x=(x_1, \cdots, x_n)$ and t.

In the present paper we shall show that the fundamental solutions of hyperbolic systems of partial differential equations with analytic coefficients are analytic except on the characteristic conoid. This property can also be expressed directly in terms of the solution of the equation: If at time $t=0$ the initial data of a solution u is analytic in an open set containing all points which lie on a ray issuing from some given point (x_0, t_0), then u is analytic at x_0, t_0.

J. Hadamard proved this property for second order hyperbolic equations [1] and M. Riesz also treated this problem [8]. In the case of constant coefficients, there are several papers which show

*) A part of this work was done while the author was a Temporary Member of the Institute of Mathematical Sciences, New York University, during the academic year 1960-1961. This Temporary Membership Plan was supported by the National Science Foundation, Contract No. NSF-G14520.