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# Reduction of models over a discrete valuation ring

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Let $\mathfrak{o}$ be a discrete valuation ring of a field $k$ and let $\mathfrak{p}$ be the maximal ideal of $\mathfrak{o}$. Then the notion of a model over $\mathfrak{o}$, defined by Nagata in [4], may be considered in a sense as a complex notion of two models defined over different fields $k$ and $\mathfrak{o}/\mathfrak{p}$ respectively. Let $M$ be a model defined over $\mathfrak{o}$. Then $M$ is a set of spots which dominate $\mathfrak{o}$ or $k$. If we denote by $M_k$ the set of all the spots in $M$ which dominate $k$, $M_k$ is an open subset of $M$ and is a model defined over $k$. On the other hand the closed subset $M-M_k$ corresponds naturally to a (not necessarily irreducible) model defined over $\mathfrak{o}/\mathfrak{p}$. Moreover each spot of $M-M_k$ is obtained as a specialization of one of $M_k$ over $\mathfrak{o}$. Then there arises naturally a question that how the structure of $M_k$ as a model over $k$ is reflected in that of $M-M_k$ as a model over $\mathfrak{o}/\mathfrak{p}$ in this specialization process over $\mathfrak{o}$. This work is initiated by this question.

On the other hand, an algebraic variety defined over $k$ is equivalent to a model defined over $k$ (cf. Chapter 1, §9 in [4]), and a theory of the reduction of algebraic varieties of any dimension with respect to a valuation $\mathfrak{p}$ of a basic field $k$ was developed by Shimura in [8]. In Shimura's theory, the reduction of a variety $V$ is naturally obtained, roughly spoken, by the reduction of defining equations for $V$, if $V$ is an affine variety or a projective variety with a "*fixed system of coordinates*". However when $V$ is an abstract variety, the reduction of $V$ depends on a choice of affine representatives of $V$. In other words it is impossible to