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Some remarks on algebraic rings

To Professor Y. Akizuki on his 60-th birthday

By

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1. In the paper [1], Greenberg called a unitary ring R to be an *algebraic ring defined over* k if the following conditions are satisfied:

1) *R is a union of a finite number of algebraic varieties defined over k.*

2) *R is an algebraic group defined over k as to its additive law.*

3) *The mapping of $R \times R$ onto R, which maps (a, b) onto ab, is an everywhere regular mapping defined over k.*

4) *The unit 1 of R is a rational point of R over k.*

5) *The set U of the units in R is a locally k-closed subset in R.*

6) *The mapping of U onto U, which maps a onto a^{-1}, is an everywhere regular mapping on U.*

In this note we shall remark, first, that if R is an algebraic ring defined over k in the above sense, then the set U of the units in R is a *k-open* subset of R, and that if the characteristic of k is zero, the conditions 5) and 6) can be excluded from the definition of an algebracic ring, i.e., if R satisfies 1), 2), 3) and 4), then R satisfies necessarily 5) and 6). Let R be an algebraic ring defined over k. Then a two-sided ideal I of R will be called an *algebric ideal* of R if I is a closed subset of R. Then we shall construct a residue class ring of R by an algebraic ideal, which is also an algebraic ring. Lastly we shall show that if R is connected, any two-sided ideal of R is a connected algebraic ideal and R is a ring with maximal and minimal conditions for two-sided ideals.

2. Let R be a unitary ring which satisfies the conditions 1), 2),