

Homotopy Groups of $SU(3)$, $SU(4)$ and $Sp(2)$

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§1. Introduction

Let $\pi_i(G)$ be the i -th homotopy group of a topological group G . For $i \leq 23$ and for $G = SU(3)$, $SU(4)$, $Sp(2)$, the groups $\pi_i(G)$ are computed and the results are given by the following table :

$i =$	3	4	5	6	7	8	9	10	11	12
$\pi_i(SU(3)) \cong$	Z	0	Z	Z_6	0	Z_{12}	Z_3	Z_{30}	Z_4	Z_{60}
$\pi_i(SU(4)) \cong$	Z	0	Z	0	Z	Z_{24}	Z_2	$Z_{120} + Z_2$	Z_4	Z_{60}
$\pi_i(Sp(2)) \cong$	Z	Z_2	Z_2	0	Z	0	0	Z_{120}	Z_2	$Z_2 + Z_2$

$i =$	13	14	15	16	17	18
$\pi_i(SU(3)) \cong$	Z_6	$Z_{84} + Z_2$	Z_{36}	$Z_{252} + Z_6$	$Z_{30} + Z_2$	$Z_{30} + Z_6$
$\pi_i(SU(4)) \cong$	Z_4	$Z_{1680} + Z_2$	$Z_{72} + Z_2$	$Z_{504} + Z_2 + Z_2 + Z_2 + Z_2$ $+ Z_2$	$Z_{40} + Z_2 + Z_2$ $+ Z_2$	$Z_{2520} + Z_{12} + Z_2$
$\pi_i(Sp(2)) \cong$	$Z_4 + Z_2$	Z_{1680}	Z_2	$Z_2 + Z_2$	Z_{40}	$Z_{2520} + Z_2$

$i =$	19	20	21	22	23
$\pi_i(SU(3)) \cong$	$Z_{12} + Z_6$	$Z_{60} + Z_6$	Z_6	$Z_{66} + Z_2$	$Z_{12} + Z_2$
$\pi_i(SU(4)) \cong$	$Z_{12} + Z_2$	$Z_{60} + Z_2$	$Z_{16} + Z_2$	$Z_{2640} + Z_4 + Z_2 + Z_2$	$Z_{24} + Z_2 + Z_2 + Z_2 + Z_2$
$\pi_i(Sp(2)) \cong$	$Z_2 + Z_2$	$Z_2 + Z_2 + Z_2$	$Z_{32} + Z_2$	$Z_{5280} + Z_2 + Z_2$	$Z_2 + Z_2 + Z_2$

These results are stated in Theorems 4.1, 5.1, 6.1, in which generators of the 2-primary components are given. The computations will be done by use of the homotopy exact sequences associated with the bundles $SU(3)/SU(2) = S^3$, $Sp(2)/Sp(1) = S^7$ and $SU(4)/SU(2) = S^5 \times S^7$ and the results [7], [3] on the homotopy