J. Math. Kyoto Univ.
7-2 (1967) 113-121

On the Structure of $H^*(BSF;\, Z_p)$

by

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(Received June 5, 1967)

§1. **Statements of Results.** The study of characteristic classes for orthogonal fibre bundles has been very useful in differential topology, differential geometry, and algebraic topology. In recent years, it has become clear that characteristic classes for PL-bundles and spherical fibre spaces will also be useful and should be studied. In this paper, we give a structure theorem for the cohomology modulo an odd prime of the classifying space for oriented spherical fibre spaces.

Let $BSF=BSG$ be the classifying space for oriented spherical fibre spaces (see [10] and [12]). $\boldsymbol{MSF}=\{MSF(n)\}$ be the associated Thom spectrum, and let[2] $\phi: H^*(BSF)\to H^*(\boldsymbol{MSF})$ be the Thom isomorphism. Let $r=2p-2$ throughout this paper. The Wu classes, $q_i\in H^{ir}(BSF)$, are defined by $q_i=\phi^{-1}(\mathcal{P}^i(\phi(1))$. Milnor [10] has shown that $H^*(BSF)$ is isomorphic to a free commutative algebra generated by q_i and βq_i (the Bockstein of q_i) in dimensions $<pr-1$. Gitler and Stasheff [5] have shown that a new element, the first exotic class, e_1, comes in dimension $pr-1$. Stasheff [13] has extended Milnor's computations and shown that q_i and βq_i generate a free commutative subalgebra of $H^*(BSF)$ in dimensions $<2pr$.

Our first theorem is the following.

THEOREM 1. a). *Let $\theta: Z_p[q_i]\otimes E(\beta q_i)\to H^*(BSF)$ be the natural map. Then θ is a monomorphism.*

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1) Partially supported by the National Science Foundation, and a Fulbright Grant.

2) All cohomology groups, unless otherwise stated, will have coefficient Z_p, p an odd prime.