On the Bott Suspension

By

I. M. JAMES

Dedicated to Professor A. Komatu on his sixtieth birthday

(Communicated by Professor H. Toda, November 5, 1968)

Introduction

In the original version [4] of the real periodicity theorem Bott introduces an operator into the structure of the homotopy groups of certain Lie groups. Following Harris [6] we refer to the operator as the Bott suspension.[1] In §1 below we give a definition of the operator which is slightly different from Bott's and clarifies its relation with the Samelson product and other constructions. Our main purpose, however, is to introduce a relative version of the Bott suspension and to study its properties.

After some consideration of the general situation in which operators of this type arise we go on to make a detailed investigation of a particular example which operates on the homotopy groups of the Stiefel manifolds

$$V_{2n,2k} = SO(2n)/SO(2n-2k) \qquad (k \leqslant n).$$

In this case the relative Bott suspension constitutes a homomorphism

$$F : \pi_r(V_{2n,2k}) \rightarrow \pi_{r+1}(V_{2n,2k}).$$

We fibre $V_{2n,2k}$ over $V_{2n,2l}$ $(l \leqslant k)$ with fibre $V_{2n-2l,2k-2l}$, in the usual way, and show that F commutes[2] with the homomorphisms in the

---

1) The precise usage of Harris is somewhat different from mine. However, the name seems to be an appropriate one for any operator of the type.
2) As usual, in this kind of situation, there will be sign changes in some cases.