J. Math. Kyoto Univ.
10-3 (1970) 403-417

Mixed problems for hyperbolic equations II

Existence Theorems with Zero Initial Datas and
Energy Inequalities with Initial Datas.

By

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(Communicated by Prof. Mizohata, March 31, 1970)

§0. Introduction

This paper has two aims. One of them is to obtain existence theorems for hyperbolic mixed problems in a quadrant:

$$(P_0)\quad \begin{cases} Au = f & \text{in } t>0,\ x>0,\ y\in R^{n-1}, \\ B_j u = g_j & (j=1, 2, \cdots, \mu) \quad \text{on } x=0,\ t>0,\ y\in R^{n-1}, \\ D_t^j u = 0 & (j=0, 1, \cdots, m-1) \quad \text{on } t=0,\ x>0,\ y\in R^{n-1}. \end{cases}$$

The other is to obtain energy inequalities for

$$(P)\quad \begin{cases} Au = f & \text{in } t>0,\ x>0,\ y\in R^{n-1}, \\ B_j u = g_j & (j=1, 2, \cdots, \mu) \quad \text{on } x=0,\ t>0,\ y\in R^{n-1}, \\ D_t^j u = u_j & (j=0, 1, \cdots, m-1) \quad \text{on } t=0,\ x>0,\ y\in R^{n-1}. \end{cases}$$

The assumption (A) stated in part I is also assumed throughout in this paper.

Our main results are as follows:

Theorem 1. *There exists a positive number γ_0 as follows. Let $f\in \mathcal{H}_{0,\gamma}((0, \infty)\times R^n_+)$, $g_j\in \mathcal{H}_{m-1-r_j,\gamma}((0, \infty)\times R^{n-1})$ $(\gamma\geqslant\gamma_0)$ be given, then there exists a unique solution u of (P_0), which belongs to $\mathcal{H}_{m-1,\gamma}((0, \infty)\times R)$.*

Theorem 2. *There exist positive numbers C and γ_0, such that it holds for $\gamma\geqslant\gamma_0$*

$$\gamma |u|^2_{m-1,\gamma,+} + \sum_{j=0}^{m-1} \langle D_x^j u\rangle^2_{m-1-j,\gamma,+}$$
$$\leqslant C\left\{\frac{1}{\gamma} |f|^2_{0,\gamma,+} + \sum_{j=1}^{\mu} \langle g_j\rangle^2_{m-1-r_j,\gamma,+} + \sum_{j=0}^{m-1} [u_j]^2_{m-1-j,\gamma}\right\}$$