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On the uniqueness of solutions of stochastic differential equations II

By

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The present paper is a continuation of [1] in which we have discussed the uniqueness of solutions of stochastic differential equations. The condition of the pathwise uniqueness of solutions obtained in [1] is essentially in one-dimensional case and we shall investigate here the general multi-dimensional case.

Let $\sigma(t, x)=(\sigma_j^i(t, x))$, $i=1, \cdots, n$, $j=1, \cdots, r$, and $b(t, x)=(b^i(t, x))$, $i=1, \cdots, n$, be defined on $[0, \infty)\times R^n$, bounded and Borel measurable in (t, x) such that $\sigma(t, x)$ is an $n\times r$-matrix and $b(t, x)$ is an $n\times 1$-matrix. We consider the following Itô's stochastic differential equation;

$$dx_t=\sigma(t, x_t)\, dB_t+b(t, x_t)\, dt, \tag{1}$$

or, in component wise,

$$dx_t^i=\sum_{j=1}^{r}\sigma_j^i(t, x_t)\, dB_t^j+b^i(t, x_t)\, dt \qquad i=1, \cdots, n. \tag{1'}$$

A precise formulation is as follows; by a probability space $(\Omega, \mathscr{F}, P)$ with an increasing family of Borel fields $\mathscr{F}_t$, which is denoted as $(\Omega, \mathscr{F}, P; \mathscr{F}_t)$, we mean a standard probability space $(\Omega, \mathscr{F}, P)$ with a system $\{\mathscr{F}_t\}_{t\in[0,\infty)}$ of sub Borel-fields of $\mathscr{F}$ such that $\mathscr{F}_t\subset\mathscr{F}_s$ if $t<s$.

Definition 1. By a solution of the equation (1), we mean a