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Residual intersections in Cohen-Macauley rings

By

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We investigate questions of the following kind: Let I be an ideal of a Cohen-Macauley local ring $(R, \mathfrak{m})$. Suppose $I=\mathfrak{a}\cap\mathfrak{b}$, where $\mathfrak{b}$ has height $\geq r$. Find conditions on I and $\mathfrak{a}$ which will imply properties of $\mathfrak{b}$. We prove among other things (2.2) that if the number of generators of I is at most r, and if $\mathfrak{a}$ is a complete intersection, then $\mathfrak{b}$ can be chosen so that $R/\mathfrak{b}$ is Cohen-Macauley. (On the other hand, if $I=\mathfrak{a}_1\cap\mathfrak{a}_2\cap\mathfrak{b}$, where $\mathfrak{a}_1$, $\mathfrak{a}_2$ are complete intersections, depth $R/\mathfrak{b}$ may be too small (cf. (2.5)). This generalizes Macauley's unmixedness theorem, which may be considered the degenerate case $\mathfrak{a}=R$.

Our method, which is quite elementary, was originally developed to answer some questions raised by Mumford about the critical locus of a map of smooth schemes. These applications are in section 5. Then, in analyzing our proof, we found that it was related to a result of Dubreil [1]. We have arranged the presentation around Dubreil's theorem.

Notation and terminology. By a local ring, we mean a noetherian local ring. Dimension (in symbol, dim) for a ring will mean the Krull dimension of the ring. Depth means, not as in [7], the length of a maximal regular sequence. We note here that if M is a finite module

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