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Note on Krull domains

by

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In this note we first generalize the theorem of Y. Mori ([6], Theorem 4, p. 27, see Corollary 1) and then we use the result to prove some well-known theorems on integral closure of a noetherian integral domain (see Corollaries 2, 3, 4). In this article, we mean by a ring a commutative ring with identity and when $\mathfrak{p}$ is a prime ideal of a ring R, we denote by $\kappa(\mathfrak{p})$ the field of quotients of $R/\mathfrak{p}$.

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Theorem. *Let R be a Krull domain. If, for any prime ideal $\mathfrak{p}$ of height one, $R/\mathfrak{p}$ is noetherian, then R is noetherian.*

Proof. Let $\mathfrak{a}$ be an ideal $\neq(0)$. Let $0\neq f\in\mathfrak{a}$. Then $(f)=\bigcap_{i=1}^{s}\mathfrak{p}_i^{(e_i)}$ with symbolic powers $\mathfrak{p}_i^{(e_i)}$ of prime ideals of height one. Therefore it is sufficient to prove the following lemma.

Lemma. *Let R be a Krull domain with field of quotients K. Let $\mathfrak{p}$ be a prime ideal of height one. If $R/\mathfrak{p}$ is noetherian, then for any natural number e, $R/\mathfrak{p}^{(e)}$ is noetherian.*

Proof. By the approximation theorem for Krull domain ([1], §1, n°. 5, Proposition 9, [3], Theorem 5. 8) we can find an element x of K such that $\nu_{\mathfrak{p}}(x)=1$ and $\nu_{\mathfrak{q}}(x)\leqq 0$ for any prime ideal $\mathfrak{q}(\neq\mathfrak{p})$ of height one. Consider the natural injection of R to $R[x]$. Then we get a natural isomorphism of $R/\mathfrak{p}$ to $R[x]/xR[x]$ (cf. [7], (11. 13), (36. 5)). By the assumption $R[x]/xR[x]$ is noetherian and therefore, $R[x]/x^eR[x]$ is noetherian by a theorem of Cohen. Now