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Asymptotic behaviours of two dimensional autonomous systems with small random perturbations

By

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0. Introduction.

Consider a following linear autonomuos system in R^2:

$$\frac{dX(t)}{dt}=\mathrm{B}\cdot X(t), \tag{0·1}$$

where B is a 2×2 constant matrix. If small linear "white noise type" perturbations act on the system $(0\cdot 1)$, we have a stochastic system:

$$dX^{\varepsilon}(t)=\mathrm{B}\cdot X^{\varepsilon}(t)\,dt+\varepsilon\{\mathrm{C}\cdot X^{\varepsilon}(t)\,dB_1(t)+\mathrm{D}\cdot X^{\varepsilon}(t)\,dB_2(t)\}, \tag{0·2}$$

where C and D are 2×2 constant matrices and $B_i(t)$ $(i=1,2)$ are independent one dimensional Brownian motions. Our interest is to study relations between properties[1] of the singular point $\{x=0\}$ of the system $(0\cdot 1)$ and of the system $(0\cdot 2)$ for sufficiently small ε.

With respect to radial parts, the relations are known, i.e., *if the origin is not a center for the system* $(0\cdot 1)$, *then*

$$\lim_{\varepsilon\to 0}\lim_{t\to\infty}|X^{\varepsilon}(t)|=\lim_{t\to\infty}|X(t)| \qquad \text{a.s.}, \tag{0·3}$$

but if the origin is a center, then the equality $(0\cdot 3)$ *is not necessarily valid.* Therefore, our purpose in this paper comes to establish such relations between an angular part $\theta(t)$ of $X(t)$ and the other one $\theta^{\varepsilon}(t)$ of $X^{\varepsilon}(t)$.

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[1] Many books, for example, Coddington and Levinson [1], discuss properties of the origin for the system (0.1).