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The conjugacy classes in the unitary, symplectic and orthogonal groups over an algebraic number field

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Introduction. Let k be a commutative field and σ be its involution, i.e. an automorphism of k such that $\sigma^2 = \text{identity}$. Let V be a symmetric bilinear (resp. symplectic; resp. Hermitian) space over k with respect to σ. Let $U(V)$ denote the group of all isometries of V. We shall concern with the conjugacy classes of the elements of $U(V)$. The problem has been studied by many mathematicians, and there are known substantial amount of results. First of all, there is a canonical injection from the set of all conjugacy classes of $U(V_0)$ for a symmetric bilinear (resp. symplectic; resp. Hermitian) space V_0 into the set of the equivalence classes of the pairs (V, x) consisting of symmetric bilinear (resp. symplectic; resp. Hermitian) spaces V and its isometries x (c.f. G. E. Wall [9] and J. Milnor [4]). The equivalence problem of the pairs (V, x) was solved by J. Williamson [10] under the assumption that the base field is perfect and of characteristic $\neq 2$, and then solved by G. E. Wall [9] under the weaker assumption called "trace condition". Then, to determine the conjugacy classes of $U(V)$ we must determine the image of the above canonical injection. The problem to determine this image can be reduced to the form of our Problem 3.3, § 3 (when k is perfect). When the base field k is finite, the latter problem is solved by G. E. Wall [9], in fact he gave an explicit description of all the conjugacy classes over finite fields. When k is a