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On Reeb components

By

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§1. Introduction

Let M be a compact orientable $n+1$ dimensional manifold and $\mathscr{F}$ a codimension one foliation on M tangent to ∂M of class C^r. $(M, \mathscr{F})$ is a *Reeb foliation* if all leaves in $\operatorname{Int} M$ are homeomorphic to $\mathbf{R}^n$. A *Reeb component* is a Reeb foliation whose leaves are proper. A Reeb foliation is always transversally orientable.

$(M, \mathscr{F})$ is *C^r conjugate* to $(M', \mathscr{F}')$ if there exists a foliation preserving C^r homeomorphism of M onto M'. $(M, \mathscr{F})$ is *C^r isotopic* to $(M, \mathscr{F}')$ if there exists a foliation preserving C^r homeomorphism of M which is C^r isotopic to the identity.

For $n=2$ Novikov ([8] for Reeb components), Rosenberg, Roussarie and Chatelet ([3], [10], [11] for Reeb foliations) has classified C^2 Reeb foliations by C^0 conjugacy and C^0 isotopy. In [6] it is shown that if $(M, \mathscr{F})$ is a Reeb foliation of class C^2 then M is homotopy equivalent to T^k (k dimensional torus) and $(M, \mathscr{F})$ is a Reeb component if and only if $k=1$.

The purpose of this note is to show that any Reeb component is an "ordinary Reeb component" if n is large. Here the *ordinary Reeb component* $(S^1 \times D^n, \mathscr{F}_R)$ (or $(S^1 \times D^n, \mathscr{F}'_R)$) is defined by $\omega = \Sigma x_i dx_i - \exp(1/(\Sigma x_i^2 - 1))dt$ where $D^n = \{(x_1, x_2, \ldots, x_n) | \Sigma x_i^2 \leqq 1\}$ and t is the coordinate of $S^1 = \mathbf{R}/\mathbf{Z}$ (or $\omega' = \Sigma x_i dx_i + \exp(1/(\Sigma x_i^2 - 1))dt$ respectively). It is easy to see that ω and ω' are completely integrable non-singular one forms on $S^1 \times D^n$ and $\mathscr{F}_R$ and $\mathscr{F}'_R$ are Reeb com-