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Cauchy problem in Gevrey classes for non-strictly hyperbolic equations of second order

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§1. Introduction

In his remarkable article [1], Y. Ohya considered the Cauchy problem for linear partial differential equations of order m which has real characteristic roots of constant multiplicity and proved its well-posedness in the Gevrey classes $\gamma_{loc}^{(a)}(1<a<m/(m-1))$ and the existence of a finite domain of dependence. There no condition is assumed on the lower order terms, which differs very much from the well-posedness in $\mathcal{E}$, cf. [9]. These facts seem to imply that Gevrey classes are suitable spaces to treat hyperbolic differential equations.

Since then the Cauchy problem in Gevrey classes has been studied in detail from various viewpoints, e.g. Leray-Ohya [2], Steinberg [4], Beals [5], Ivriĭ [6], etc. However we should remark the followings. In [1], [2], [4], the smoothenss of the characteristic roots play an essential role. In [5], [6], the smoothness of the characteristic roots is not assumed, but it is assumed in [5] that the coefficients do not depend on time variable t and also that the characteristic roots do not vanish, and in [6] that the coefficients of the principal part of the differential operator are analytic.

Now we state our result. Consider the partial differential equation of second order

$$L[u]=\delta^2 u-\partial_i(a^{ij}\partial_j u)-b^i\partial_i u-cu=f(x, t), \tag{1.1}$$

$(x, t)\in\Omega=\boldsymbol{R}^n\times[0, h]$, $h>0$, where $\delta=\partial_t+a^i\partial_i+b^0$, $a^{ij}(x, t)=a^{ji}(x, t)$, it is supposed that repeated indices are summed from 1 to n, e.g. $\partial_i(a^{ij}\partial_j u)=\sum_{i,j=1}^n \partial_i(a^{ij}\partial_j u)$[1].

Definition 1.1. $(\gamma_{loc}^{(a)}, \gamma^{(a)}, \gamma_0{}^{(a)})$.

We say that $\phi(x)\in\mathcal{E}$ *belongs to* $\gamma_{loc}^{(a)}$ *if for any compact set* K, *there exist*

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1) Throughout this paper, we use the following abbreviations and function spaces: $x=(x_1, x_2, \cdots, x_n)$, $\xi=(\xi_1, \xi_2, \cdots, \xi_n)$, $p=(p_1, p_2, \cdots, p_n)$; p_i are non-negative integers, $|p|=p_1+p_2+\cdots+p_n$, $e_i=(0, \cdots, 1, \cdots, 0)$, $\partial_t=\partial/\partial t$, $\partial_i=\partial/\partial x_i$, $\partial^p=\partial_1{}^{p_1}\partial_2{}^{p_2}\cdots\partial_n{}^{p_n}$, $\partial^p\phi(x)=\phi_{(p)}(x)$, $\partial_i\phi(x)=\{\phi(x)\}'_{x_i}$, $(u, v)=\int_{\boldsymbol{R}^n} u(x)v(x)dx$, $\|u\|^2=\int_{\boldsymbol{R}^n}|u(x)|^2dx$.

$\phi\in\mathcal{E}$ means that ϕ is an infinitely differentiable function, $\phi(x)\in\mathcal{D}_{L^2}{}^{\infty}$ means that $\phi(x)$ and all of its derivatives (in the distribution sense) are square integrable. $\phi(x, t)\in\mathcal{D}_{L^2}{}^{\infty}[0, h]$ means that $t\to\phi(x, t)\in\mathcal{D}_{L^2}{}^{\infty}$, $0<t<h$, is infinitely differentiable, cf. [8].