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# Automorphic forms and algebraic extensions of number fields, II

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## Introduction

This is a continuation of the previous papers [7], [8] with the same title. Let $F$ be a totally real algebraic number field which is a prime cyclic extension of the rational number field $\boldsymbol{Q}$ and satisfies the conditions $1\sim3$ in §1, $\mathfrak{o}$ its maximal order and $\sigma$ a generator of the Galois group $\mathrm{Gal}(F/\boldsymbol{Q})$. In [7], [8], we defined a subspace $\mathcal{S}_{\kappa}(SL_2(\mathfrak{o})^+)$ of the space $S_{\kappa}(GL_2(\mathfrak{o})^+)$ of Hilbert cusp forms of weight $\kappa$ with respect to $GL_2(\mathfrak{o})^+$ by means of an action $T_{\sigma}$ of $\sigma$ and Hecke operators on $S_{\kappa}(GL_2(\mathfrak{o})^+)$, and gave the traces of Hecke operators on this subspace by using a twisted trace formula on $S_{\kappa}(GL_2(\mathfrak{o})^+)$. Moreover we showed the identity between the twisted trace formula and the ordinary trace formula on spaces of cusp forms of one variable, and using this identity we proved a generalization of Doi-Naganuma's result [1], [6] on lifting of cusp forms. In this paper, we shall generalize the above result to the case of congruence subgroups $\Gamma_0(\mathfrak{n})$ with some integral ideal $\mathfrak{n}$ of $F$. For an integral ideal with ${}^{\sigma}\mathfrak{n}=\mathfrak{n}$, we can define a subspace $\mathcal{S}_{\kappa}(\Gamma_0(\mathfrak{n}))$ of $S_{\kappa}(\Gamma_0(\mathfrak{n}))$ in the similar way, and can calculate the traces of Hecke operators on this subspace by using a twisted trace formula (Theorem 4.2). As in the above case, we can show the identity between the twisted trace formula and the ordinary trace formula for Hecke operators for spaces of cusp forms of one variable. By virtue of this identity, we can generalize the above result on lifting of cusp forms in the case of congruence subgroups.

Our result has been generalized in adelic and representation-theoretic setting by Shintani [12] and Langlands [4]. But we think it is not meaningless to give an explicit result in the classical case.

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## Notation

The symbols $\boldsymbol{Z}$, $\boldsymbol{Q}$, $\boldsymbol{R}$ and $\boldsymbol{C}$ denotes respectively the ring of rational integers, the rational number field, the real number field and the complex number field.