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Derivatives of Wiener functionals and absolute continuity of induced measures

By

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1. Introduction

The *Wiener space*, which is a typical example of abstract Wiener spaces introduced by L. Gross [1], is a triple (B, H, μ) where
(i) B is a Banach space consisting of real-valued continuous functions $x(t)$ with $x(0)=0$ defined on the interval $T=[0, 1]$ endowed with norm $\|x\|=\sup_{0\le t\le 1}|x(t)|$,
(ii) H is a Hilbert space consisting of absolutely continuous functions $x(t)$ with $x(0)=0$ such that $x'(t)\in L^2(T)$ endowed with the inner product

$$\langle x, y\rangle_H=\int_0^1 x'(t)y'(t)dt$$

and
(iii) μ is the Wiener measure, i.e., the Borel probability measure on B such that

$$\int_B \exp\{i(h, x)\}\mu(dx)=\exp\left(-\frac{1}{2}\langle h, h\rangle_H\right) \tag{1.1}$$

where $h\in B^*\subset H$ and $(\ ,\ )$ is a natural pairing of B^* and B. Note that $\|x\|\le|x|_H=\sqrt{\langle x, x\rangle_H}$ for $x\in H$, then the inclusion map $i: H\to B$ is continuous. Hence we have $B^*\subset H^*=H$ and we regard B^* as a subset of H. It is readily seen that $\{x(t); 0\le t\le 1\}$ is a standard Wiener process on the probability space (B, μ). A real-valued (or more generally, a Banach space-valued) measurable function defined on the probability space (B, μ) is called a *Wiener functional*. We identify two Wiener functionals $F_1(x)$ and $F_2(x)$ if $F_1(x)=F_2(x)$ a.e. (μ). Typical examples of Wiener functionals are solutions of stochastic differential equations or Ito's multiple Wiener integrals [2].

P. Malliavin introduced a notion of derivatives of Wiener functionals and applied it to the absolute continuity and the smoothness of density of the probability law induced by the solution of the stochastic differential equation at a fixed time [6], [7]. Here we define the derivatives of the Wiener functionals in a somewhat