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Simply connected compact simple Lie group $E_{6(-78)}$ of type E_6 and its involutive automorphisms

Dedicated to professor Tatsuji Kudo for his 60th birthday

By

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It is known that there exist five simple Lie groups of type E_6 up to local isomorphism, one of them is compact and the others are non-compact. One of the non-compact Lie groups of type E_6 is obtained by

$$E_{6(-26)} = \{\alpha \in \mathrm{Iso}_{\boldsymbol{R}}(\mathfrak{J}, \mathfrak{J}) \mid \det \alpha X = \det X\}$$

where $\mathfrak{J}$ is the exceptional Jordan algebra over the field of real numbers $\boldsymbol{R}$ [1]. In this paper, we consider the compact case. The first results are as follows. The simply connected compact simple Lie group of type E_6 is explicitly given by

$$E_6 = \{\alpha \in \mathrm{Iso}_{\boldsymbol{C}}(\mathfrak{J}^C, \mathfrak{J}^C) \mid \det \alpha X = \det X, \quad \langle \alpha X, \alpha Y \rangle = \langle X, Y \rangle\}$$

where $\mathfrak{J}^C$ is the split exceptional Jordan algebra over the field of complex numbers $\boldsymbol{C}$ and $\langle X, Y \rangle$ the positive definite Hermitian inner product in $\mathfrak{J}^C$. The center $z(E_6)$ of the group E_6 is

$$Z_3 = \{1, \omega 1, \omega^2 1\}, \qquad \omega \in \boldsymbol{C}, \ \omega^3 = 1, \ \omega \neq 1.$$

It is also known that the group E_6 has four involutive automorphisms (up to inner automorphism in the automorphism group of E_6)

$$\boldsymbol{\tau}, \qquad \boldsymbol{\sigma}, \qquad \boldsymbol{\gamma}, \qquad \boldsymbol{\rho},$$

and types of the groups corresponding to them are respectively

$$F_4, \qquad D_1 \oplus D_5, \qquad C_1 \oplus A_5, \qquad C_4.$$