J. Math. Kyoto Univ. (JMKYAZ)
21-2 (1981) 231-238

On Rees algebras of ideals generated by a subsystem of parameters

By

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(Received July 31, 1979)
(Revised January 11, 1980)

1. Introduction.

The purpose of this paper is to prove the following

Theorem 1.1. *Let A be a Noetherian local ring and let $0<r<\dim A$ be an integer. Then the following conditions are equivalent.*
(1) *A is a Cohen-Macaulay ring.*
(2) *The Rees algebra $R(q)=\bigoplus_{n\geq 0} q^n$ is a Cohen-Macaulay ring for every ideal q of A generated by a subsystem of parameters for A of length r.*

In case A is a Cohen-Macaulay local ring J. Barshay [1] showed that the Rees algebras of ideals generated by subsystems of parameters for A are always Cohen-Macaulay (cf. p. 93, Corollary), and it seems to be natural to ask whether the converse of his result is true. But unfortunately this does not hold in the case of the parameter ideals. In fact, recently S. Goto and the author [2] (cf. Theorem 1.1), heve proved that the Rees algebras of parameter ideals of certain Buchsbaum local rings are always Cohen-Macaulay. Neverthless the above theorem guarantees that the converse of Barshay's result is true if the length of subsystems of parameters considered is less than dim A.

The idea of the proof of Theorem 1.1 is essentially same as that of the proof of the main theorem of [2]. We will prove Theorem 1.1 in Section 3. Section 2 will be devoted to some preliminary results which we shall need for this purpose. Finally we will show with an example that the condition that *every* ideal q of A generated by a subsystem of parameters for A is not superfluous.

In this paper we denote by A a Noetherian local ring of dimension d with maximal ideal m.

2. Preliminary.

Let q be an ideal of A generated by a subsystem of parameters for A of length r. We put $q=(a_1, a_2, \cdots, a_r)$ and $R=R(q)$. Notice that the ring R is