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On boundedness of families of torsion free sheaves

By

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Dedicated to Professor Y. Nakai on his sixties birthday

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Introduction

In [8] Kleiman has developed an effective method of deciding whether a family of coherent sheaves is bounded or not. And then he gave an interesting result on families of invertible sheaves:

Theorem ([8] Theorem 3.13). Let *S be a noetherian scheme and X a projective S-scheme with S-ample invertible sheaf $\mathcal{O}_X(1)$. Suppose that the geometric fibres of X/S are integral and of dimension n. Then, for a family $\mathfrak{F}$ of the classes of invertible sheaves on the fibres of X/S, the following conditions are equivalent*:

(i) *$\mathfrak{F}$ is bounded.*

(ii) *In the Hilbert polynomial $\chi(L(m)) = \sum_{i=0}^{n} a_i \binom{m+n-i}{n-i}$ of $L \in \mathfrak{F}$, the coefficient a_1 is bounded and a_2 is bounded below.*

(iii) *Whenever L runs over $\mathfrak{F}$, the degree $d(c_1(L), \mathcal{O}_X(1))$ is bounded and $d(c_1(L)^2, \mathcal{O}_X(1))$ is bounded below.*

On the other hand, thanks to [12] Corollary 5.9.1, the boundedness of semi-stable sheaves is equivalent to the projectivity of moduli spaces of semi-stable sheaves. In [13] the author proposed that the stronger statements $B_{n,r}(\Lambda)$ and $B'_{n,r}(\Lambda)$ should be proved and showed that in some special cases they held. Later H. Spindler [16] found that the technique of Barth ([2] and [13] § 4) could be generalized for every rank when the characteristic of the base field was zero. And then, as was indicated in [16], it is easy to see that $B''_{n,r}(\Lambda)$ is true for all n and r whenever Λ is a field of characteristic zero (see also [5]). However, once one tries to generalize Kleiman's theorem stated in the above, there is no reason for sticking to semi-stable sheaves, much less doing to $B''_{n,r}(\Lambda)$.

This article is concerned with a generalization of Kleiman's theorem in the forms which contain $B_{n,r}(\Lambda)$, $B'_{n,r}(\Lambda)$ and hence $B''_{n,r}(\Lambda)$ as their special