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Vanishing of $\mathrm{Ext}_A^i(M, A)$

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1. Let A denote a Noetherian ring. The purpose of this note is to establish a kind of vanishing theorem on $\mathrm{Ext}_A^i(M, A)$ over Gorenstein rings. Our result is

Theorem 1. *The following conditions are equivalent.*

(1) *A is a Gorenstein ring.*
(2) *For every finitely generated A-module M there exists an integer n depending on M such that*

$$\mathrm{Ext}_A^i(M, A)=(0)$$

for all $i\geqq n$.

In case A has finite Krull-dimension, say d, it is well-known by Bass [2] that A is a Gorenstein ring if and only if $\mathrm{Ext}_A^i(M, A)=(0)$ for every finitely generated A-module M and for every integer $i>d$. This doesn't make sense if A has infinite Krull-dimension and of course our theorem remains valid even in this case.

In their lecture [1] Auslander and Bridger introduced the concept of Gorenstein-dimension of finitely generated modules and gave a characterization of Gorenstein local rings (and hence of Gorenstein rings with finite Krull-dimension) in terms of Gorenstein-dimension. By virtue of Theorem 1 we can easily extend their result to an assertion about arbitrary Noetherian rings:

Corollary 2. *A is a Gorenstein ring if and only if every finitely generated A-module has finite Gorenstein-dimension.*

As a direct consequence of this fact we have the following

Corollary 3. *A is a regular ring if and only if every finitely generated A-module has finite projective dimension.*

2. First we note

Lemma 4. *Let*

$$0 \longrightarrow M_1 \longrightarrow M_2 \longrightarrow M_3 \longrightarrow 0$$