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On certain hermitian symmetric submanifolds of bounded domains

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Introduction.

Let M be a Kähler manifold and $p \in M$. In [9], the author and Y. Se-ashi constructed a totally geodesic hermitian symmetric submanifold $M(p)$ of M through p satisfying certain conditions. A bounded domain in C^n is always Kähler manifold with the Bergmann metric. Thus we can apply the above result to D and obtain the hermitian symmetric submanifold $D(p)$ of the non-compact type. The purpose of this paper is to study $D(p)$ for a bounded domain D satisfying the condition (A.1) or $(A.2)_p$. Let $\mathfrak{g}(D)$ be the Lie algebra of Aut(D) and let $\mathfrak{g}(D)_c$ be its complexification. Then (A.1) implies that $\mathfrak{g}(D)_c$ is "transitive" and $(A.2)_p$ says the existence of an element in the isotropy subalgebra of $\mathfrak{g}(D)_c$ at p which is mapped to the identity transformation of $T_p(D)$ by the isotropy representation. For example, every bounded domain which is equivalent to a tube domain satisfies (A.1) and every bounded circular domain containing the origin 0 satisfies $(A.2)_0$. Furthermore every bounded domain which is equivalent to a Siegel domain of the second kind satisfies (A.1) and $(A.2)_p$ for each point p. In particular every homogeneous bounded domain also satisfies (A.1) and $(A.2)_p$ for each point p.

Our main results are the followings:

(a) *Assume that a bounded domain D satisfies* (A.1). *Then for any $p, p' \in D$, $D(p)$ and $D(p')$ are holomorphically isomorphic to each other* (*Theorem* 3.3).

(b) *Assume that a bounded domain D satisfies* $(A.2)_p$. *Then $D(p)$ is a maximal hermitian symmetric submanifold of D through p in a certain sence* (*Theorem* 4.4).

Let D be a Siegel domain of the second kind. Then D is holomorphically equivalent to a bounded domain. In connection with the study of the non-affine part of $\mathfrak{g}(D)$, the author constructed a symmetric Siegel domain S associated with D and realized D as a Siegel domain of the third kind with the base space S ([6], [7]). We can see that our symmetric space $D(p)$ is nothing but the symmetric Siegel domain S.

Throughout this paper, we shall use the following notations: Aut(D) means the group of all holomorphic transformations of a complex manifold D which is equivalent to a bounded domain and Aut$^0(D)$ denotes its identity component. For a real vector space or a real Lie algebra W, W_c means its complexification and for $w \in W_c$, $\overline{w}$ denotes