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Some remarks on meromorphic functions on open Riemann surfaces

Dedicated to Professor Yukio Kusunoki on his sixtieth birthday

By

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Introduction. The generalization of Abel's theorem to open Riemann surfaces in the form analogous to the classical one has been studied by many authors (for recent references, Maitani [4], Sainouchi [6], [7], Watanabe [8]). In this paper, we define a class $\mathscr{M}$ ($\mathscr{M}_0$) of meromorphic functions on an arbitrary open Riemann surface (for the definition of $\mathscr{M}$ ($\mathscr{M}_0$), see section **2**) and give a necessary and sufficient condition for the existence of a meromorphic function of $\mathscr{M}$ ($\mathscr{M}_0$) which has the given divisor. It has a similar formulation to classical Abel's theorem, but we do not assume the finiteness of divisor. In the last section, for certain class of Riemann surfaces with a metric condition, we give some sufficient conditions in order that a meromorphic function should belong to the prescribed class.

1. We shall consider an arbitrary open Riemann surface R and denote its genus by g $(0<g\leqq+\infty)$. Let $\{\Omega_n\}$ $(n=1, 2,\dots)$ be a canonical exhaution of R, then there exists a canonical homology basis $\{A_i, B_i\}$ $(i=1, 2,\dots, k(n),\dots)$ with respect to $\{\Omega_n\}$ such that $\{A_i, B_i\}$ $(i=1, 2,\dots, k(n))$ for a canonical homology basis of Ω_n (mod $\partial\Omega_n$). Let $\mathscr{D}$ be a class of an enumerable number of semiexact holomorphic differentials dw_i $(i=1, 2,\dots)$ on R such that $\int_{A_j} dw_i=\delta_{ij}$ (Kronecker's δ) and set $\int_{B_j} dw_i=B_{ij}=\xi_{ij}+i\tau_{ij}$ (ξ_{ij}, τ_{ij}; real). Also a class of the square integrable semiexact holomorphic differentials having the same property as above is denoted by $\mathscr{D}_0$. The class $\mathscr{D}$ ($\mathscr{D}_0$) always exists on an arbitrary open Riemann surface, but does not be determined uniquely for given canonical homology basis.

2. Let $\mathscr{M}$ be a class of meromorphic functions on R such that each function f belonging to $\mathscr{M}$ has the following two properties: (1) there exists an integer n_0 such that for all n $(\geqq n_0)$

$$\int_{\gamma_n^{(i)}} d\log f=0 \qquad (i=1, 2,\dots, m_n),$$

where $\gamma_n^{(i)}$ are the components of $\partial\Omega_n$ and do not contain the zeros and poles of f.

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