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On factorization of certain entire and meromorphic functions

Dedicated to Professor Yukio Kusunoki on his 60th birthday

By

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Introduction.

In the previous paper [20], we investigated the problem concerning the uniqueness of the factorization (under composition) of certain entire functions, as well as the primeness etc. And in [22], we introduced the notion of primeness in divisor sense (for entire functions) and studied about it.

In this paper, using results obtained in [20] and [22], we shall deal with some related problems on factorization of certain entire and meromorphic functions (which are closely related to periodic functions). Among others, we are mainly concerned about entire functions which belong to $\boldsymbol{J}(b)$ or $\boldsymbol{L}(b)$ (cf. § 1). For instance, an entire function $f(z)$ belongs to $\boldsymbol{J}(b)$ $(b\neq 0)$, if $f(z)$ can be expressed as $f(z)=cz+H(z)$, where c is a non-zero constant and $H(z+b)\equiv H(z)$ is, periodic, entire.

In § 1, we recall some definitions, terminologies and several known facts, needed later. In § 2, we will generalize and complement the former results on $\boldsymbol{J}(b)$ and $\boldsymbol{L}(b)$. § 3 contains a result on the deficiency and factorizability, relating to the work due to Gross-Osgood-Yang [9] (originally Goldstein [4]). In § 4, we shall consider a factorization problem concerning the (iterative) functional equation: $f\circ f=g\circ g$, and prove that certain entire functions f and g satisfying this equation are identical (see Theorem 6). In § 5, applying a result on primeness in divisor sense, we shall exhibit certain meromorphic functions which are prime.

§ 1. Preliminaries.

1.1. Definitions and terminologies. For a meromorphic function $F(z)$, the factorization under composition operation such as

$$F(z)=f\circ g(z)=f(g(z)) \tag{1}$$

has been considered, where f and g are meromorphic functions. Of course, when f is transcendental, then g should be entire. Then, by definition ([5]), F is called to be *prime* (*pseude-prime*; *right-prime*; *left-prime*), if, for every factorization as above, we can always deduce the following assertion: f or g

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