Some special cases of a conjecture of Sharp

Dedicated to Professor Masayoshi Nagata on his sixtieth birthday

By

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Introduction.

The purpose of this paper is to discuss a conjecture of Sharp on the existence of dualizing complexes:

Sharp's Conjecture ([30, 4.4]). A ring with a dualizing complex is a homomorphic image of a Gorenstein ring.

We have succeeded in giving an affirmative answer to the conjecture above in some special cases for local rings only. However the results are interesting, and further, the methods used attracted us and seem to point to one of the directions in commutative algebra. Therefore the writers would like to release this paper.

The notion of a dualizing complex was introduced by Grothendieck and Hartshorne to extend the duality theory to a large class of schemes and rings ([20]). The duality theory has a long history, and has been and will continue to be one of the most important themes in mathematics. We now have many versions of the duality theory in commutative algebra and algebraic geometry: for example, Macaulay's inverse systems ([22]), Serre's duality theory, Grothendieck's duality theory (cf. [1]), Matlis' duality theory ([23]), Grothendieck's local duality theory ([14], [15]), Herzog-Kunz's theory of the canonical module ([21]), Goto-Watanabe's duality theory for graded rings ([12]), and the theory of dualizing complexes ([20]). At the present time the theory of dualizing complexes seems to offer the vastest version of the duality theory in commutative algebra and algebraic geometry. In recent years it has become clear that the theory of dualizing complexes is very useful and powerful in commutative algebra, and some important results are shown by using it (cf. Introductions of [28], [30] and [6]). In regard to the existence of dualizing complexes, some necessary or sufficient conditions are known (cf. § 1, [8], [25], [33]). Classical-algebraic-geometric rings have dualizing complexes. However the class of rings with dualizing complexes is not yet clarified. Sharp's conjecture asserts that this class of rings coincides with the class of rings which are homomorphic images of finite-dimensional Gorenstein rings. In this paper we shall tackle this