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Necessary and sufficient conditions for the local solvability of the Mizohata equations

By

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§ 1. Introduction.

As is well known, there exists a suitable $C^\infty(R^2)$ function $f(x_1, x_2)$ such that the Mizohata equation

$$(1.1) \qquad M_n u(x_1, x_2) \equiv \frac{\partial u}{\partial x_1} + i x_1^{2n+1} \frac{\partial u}{\partial x_2} = f(x_1, x_2)\,,$$

where n is a non-negative integer, does not have a distribution solution in any neighborhood of the origin. But it seems the necessary and sufficient conditions on $f(x_1, x_2)$ for (1.1) to have a local solution are not yet known except for those of the micro-local solvability (see Sato-Kawai-Kashiwara [6] and Hörmander [2]).

In this article, we are concerned with the necessary and sufficient conditions on $f(x_1, x_2)$ for (1.1) to have a C^1 solution in a neighborhood of the origin.

Definition. We say a function $f(x_1, x_2)$ is the admissible data for the local solvability of (1.1) at the origin when (1.1) has a C^1 solution in a neighborhood of the origin.

Let Ω and $\mathcal{I}$ denote respectively an open neighborhood of the origin in R^2 and an open interval $(-r, r)$. Throughout this article m denotes $2n+2$. Now, our main result is stated thus:

Theorem A. *Assume that $f(x_1, x_2) \in C^0(\Omega)$ and $\partial_{x_2} f(x_1, x_2)$ is Hölder continuous in Ω. Let $f^\sharp(x_1, x_2)$ denote the function defined in Ω by*

$$\int_{-x_1}^{x_1} \partial_{x_2} f(t, x_2)\,dt\,.$$

Then, $f(x_1, x_2)$ is the admissible data for the local solvability of (1.1) *at the origin if and only if there exists a positive constant δ such that the function $A_m^\sharp f(x_2)$ defined in R^1 by*

$$\int_{-\delta}^{\delta} \int_0^{\delta} \frac{f^\sharp((m y_1)^{1/m}, y_2)}{y_1 + i(y_2 - x_2)}\,dy_1\,dy_2$$

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