J. Math. Kyoto Univ. (JMKYAZ)
29-1 (1989) 1-16

Ample vector bundles of small c_1-sectional genera

By

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Introduction

Let $\mathscr{E}$ be a vector bundle of rank r on a compact complex manifold M of dimension n. Let $P = \mathbf{P}(\mathscr{E})$ be the associated $\mathbf{P}^{r-1}$-bundle and let $H = H(\mathscr{E})$ be the tautological line bundle $\mathcal{O}(1)$ on P. $\mathscr{E}$ is said to be ample if so is H on P. In this case $A = \det(\mathscr{E})$ is also ample on M. The c_1-sectional genus g of $\mathscr{E}$ is defined to be $g(M, A)$, which is determined by the formula $2g(M, A) - 2 = (K + (n-1)A)A^{n-1}$, where K is the canonical bundle of M. Then $g(M, A)$ is a non-negative integer by [F5]. In this paper we establish a classification theory of the case $g(M, A) \leqq 2$. The case $r = 1$ was treated in [F6] and we study here the case $r > 1$. In §1, we study the case $g = 0$ or 1. The case $g = 2$ is studied in §2. The main theorem is in (2.25). In §3, we give a classification according to the sectional genus of (P, H).

The author thanks Professor S. Mukai for helpful comments during the preparation of this paper.

We employ similar notation to that in our previous papers on polarized manifolds.

§1. The case $g \leqq 1$

(1.1) Throughout this paper let $\mathscr{E}$, M, P, H, A and K be as in the introduction. We further assume that $n \geqq 2$ and $r \geqq 2$.

(1.2) The canonical bundle K^P of P is $\pi^*(K + A) - rH$, where π is the projection $P \to M$. So $2g(P, H) - 2 = (K^P + (n + r - 2)H)H^{n+r-2} = (n-2)H^{n+r-1} + H^{n+r-2}\pi^*(K + A) = (n-2)s_n(\mathscr{E}) + (K + A)s_{n-1}(\mathscr{E})$, where $s_j(\mathscr{E})$ is the j-th Segre class of $\mathscr{E}$. The total Segre class $s(\mathscr{E})$ is related to the Chern class by the formula $s(\mathscr{E})c(\mathscr{E}^\vee) = 1$. Thus, in particular, we have $2g(P, H) - 2 = (K + A)A = 2g(M, A) - 2$ and $g(P, H) = g(M, A)$ in case $n = 2$.

Remark. If M were a curve, both $g(P, H)$ and $g(M, A)$ would be equal to the genus of M. However, if $n \geqq 3$, $g(P, H) \neq g(M, A)$ in general.

(1.3) Lemma. *$AC \geqq r$ for any rational curve C in M.*

Communicated by Prof. Nagata, Dec. 12, 1986