J. Math. Kyoto Univ. (JMKYAZ)
31-1 (1991) 71-82

On the structure of infinitesimal automorphisms of linear Poisson manifolds I

Dedicated to Professor Noboru Tanaka on his sixtieth birthday

By

Nobutada NAKANISHI

Introduction

Let M be a smooth manifold. A Poisson structure on M is defined as a Lie algebra structure $\{\cdot, \cdot\}$ on $C^\infty(M)$ satisfying Leibniz identity. Let $x_1, x_2, \ldots, x_n$ be local coordinates on M. Then as is usual [6], this is equal to giving an antisymmetric contravariant 2-tensor P on M which satisfies Jacobi identity. In the local coordinates expression, P satisfies:

$$P = \frac{1}{2} \sum_{1 \leqslant i,j \leqslant n} P_{ij} \frac{\partial}{\partial x_i} \wedge \frac{\partial}{\partial x_j}, \quad \text{with } P_{ij} = -P_{ji}, \tag{0.1}$$

$$\sum_{1 \leqslant \ell \leqslant n} \left(P_{i\ell} \frac{\partial P_{jk}}{\partial x_\ell} + P_{j\ell} \frac{\partial P_{ki}}{\partial x_\ell} + P_{k\ell} \frac{\partial P_{ij}}{\partial x_\ell} \right) = 0, \quad \text{for } 1 \leqq i, j, k \leqq n. \tag{0.2}$$

The corresponding Lie algebra structure on $C^\infty(M)$ is called a Poisson structure on M.

Next we shall define here a linear Poisson manifold, which is one of the most important examples of Poisson manifolds. Let G be a connected Lie group whose Lie algebra is $\mathfrak{g}$. Let $\mathfrak{g}^*$ be the dual space of $\mathfrak{g}$. If $x_1, x_2, \ldots, x_n$ is a basis of $\mathfrak{g}$ satisfying

$$[x_i, x_j] = \sum_{k=1}^{n} c_{ijk} x_k, \tag{0.3}$$

then from this bracket operation, we can define the Poisson bracket $\{\cdot, \cdot\}$ on $C^\infty(\mathfrak{g}^*)$ as follows:

$$\{f, g\} = \sum_{1 \leqslant i,j,k \leqslant n} c_{ijk} x_k \frac{\partial f}{\partial x_i} \cdot \frac{\partial g}{\partial x_j}, \tag{0.4}$$

where $C^\infty(\mathfrak{g}^*)$ denotes an algebra of C^∞-function on $\mathfrak{g}^*$. Note that each x_k is considered as a linear function on $\mathfrak{g}^*$. By this Poisson bracket, $C^\infty(\mathfrak{g}^*)$ becomes a

---

Communicated by Prof. H. Toda Feb. 1, 1989
Revised May 31, 1990