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Generating functions and integral representations for the spherical functions on some classical Gelfand pairs

By

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Introduction

Let $\mathbf{F}$ be $\mathbf{R}$, $\mathbf{C}$ or $\mathbf{H}$ and $a \mapsto \bar{a}$ the usual conjugation in $\mathbf{F}$. We define the following quadratic form in $\mathbf{F}^{n+1}$.

$$(x, y)_- = -\bar{x}_0 y_0 + \bar{x}_1 y_1 + \cdots + \bar{x}_n y_n .$$

Let $U(1, n; \mathbf{F})$ be the group of the linear transformations g in $\mathbf{F}^{n+1}$ which satisfy $(gx, gy)_- = (x, y)_-$ for all x, $y \in \mathbf{F}^{n+1}$. We define the group G as follows.

1. If $\mathbf{F} = \mathbf{R}$, G is the connected component of the unit element in $U(1, n; \mathbf{R})$, i.e. $G = SO_0(1, n)$.
2. If $\mathbf{F} = \mathbf{C}$, G is the group of all the elements $g \in U(1, n; \mathbf{C})$ of determinant one, i.e. $G = SU(1, n)$.
3. If $\mathbf{F} = \mathbf{H}$, $G = U(1, n; \mathbf{H})$, i.e. $G = Sp(1, n)$.

Let $B(\mathbf{F}^n)$ be the unit ball in $\mathbf{F}^n$ and $S(\mathbf{F}^n)$ be the unit sphere in $\mathbf{F}^n$. The group G acts transitively on $B(\mathbf{F}^n)$ and $S(\mathbf{F}^n)$ as follows: for $\xi = {}^t(\xi_1, \ldots, \xi_n) \in \mathbf{F}^n$ and $g = (g_{pq})_{0 \le p,q \le n} \in G$, we define

$$\xi' = g\xi ,$$

where $\xi' = {}^t(\xi'_1, \ldots, \xi'_n)$, with

$$\xi'_p = \left(g_{p0} + \sum_{q=1}^{n} g_{pq}\xi_q\right)\left(g_{00} + \sum_{q=1}^{n} g_{0q}\xi_q\right)^{-1}, \qquad 1 \le p \le n .$$

Let K be the isotropy group of $O \in B(\mathbf{F}^n)$ in G. Then K is a maximal compact subgroup of G and $G/K \cong B(\mathbf{F}^n)$. Let $G = KAN$ be the corresponding Iwasawa decomposition and M be the centralizer of A in K. Then M is the isotropy group of $e_1 = {}^t(1, 0, \ldots, 0) \in S(\mathbf{F}^n)$ in K and $K/M \cong S(\mathbf{F}^n)$ is the Martin boundary on $G/K \cong B(\mathbf{F}^n)$. Except for the case of real numbers, K/M is not a symmetric space, but it is known that (K, M) is a Gelfand pair, i.e. the convolution algebra of functions on K bi-invariant by M is commutative. As is well known, the spherical functions on K/M play an important role in the harmonic analysis on G/K.

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