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On the initial-boundary value problems for barotropic motions of a viscous gas in a region with permeable boundaries

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1. Introduction

The one-dimensional motion of a viscous polytropic gas is described by the following system of equations [1], [13]:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial p}{\partial y}\ , \tag{1.1}$$

$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial y}+\rho\frac{\partial u}{\partial y}=0\ , \tag{1.2}$$

$$c_v\rho\left(\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial y}\right)=\kappa\frac{\partial^2 \theta}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2-p\frac{\partial u}{\partial y}\ . \tag{1.3}$$

The system is a simplified form of the Navier-Stokes equations. Here u, ρ, θ and p are the velocity, density, absolute temperature and pressure, respectively — the required characteristics of the medium; y is the Cartesian coordinate; t is the time; μ, c_v, κ are the viscosity, specific heat capacity and thermal conductivity — positive constants.

The system is supplemented with the equation of state

$$p=p(\rho,\ \theta) \tag{1.4}$$

We have a closed set of the equations of an ideal (perfect) gas if the equation of state takes the form

$$p=R\rho\theta\ ,$$

where R is the universal gas constant.

The model called the generalized Burgers' equations of viscous gas is defined by the simplest equation of state:

$$p=\text{const}>0\ .$$

In our paper the main attention will be paid to the equations of a barotro-

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