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Stationary measures for automaton rules 90 and 150

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This is a continuation of [3]. Let $\Omega=\{0,+1\}^{Z}$. A transformation Λ: $\Omega\rightarrow\Omega$ is defined as follows;

$$\Lambda x\ (i)=x\ (i-1)+x\ (i+1) \bmod 2\ ,$$

where $x\in\Omega$ and $i\in Z$. In [3] Λ was called *one-dimensional life game*. According to the classification of one-dimensional automata by Wolfram [5], this is rule 90. We are interested in the Λ-invariant measures on Ω. For $0\leqq p\leqq 1$, let β_p be the distribution of the Bernoulli trials with density p. It is shown in [2,3,4] that $\beta_{1/2}$, the distribution of coin tossing, is Λ-invariant.

Furthermore, let M be the set of translation-invariant mixing measures on Ω and let Conv (M) be the convex hull of M, i.e., the set of convex combinations of measures in M. If we replace the adjective "mixing" with "ergodic", we have the set Conv (E) of all translation-invariant measures (the ergodic decomposition theorem). The behaviour of $\Lambda^n P$ as $n\rightarrow\infty$ for $P\in$ Conv (M) is quite different from that for $P\in$ Conv $(E)\setminus$Conv (M). First we see the behaviour for $P\in$ Conv (M). The following theorem is an improvement of Theorem 3 in [3].

Theorem 1. *Assume $P\in$ Conv (M). Then, $\Lambda^n P$ converges as $n\rightarrow\infty$ if and only if P is a convex combination of β_0, $\beta_{1/2}$ and β_1.*

Collorary (Theorem 1 in [3]). *Assume $P\in$ Conv (M). P is Λ-invariant if and only if P is a convex combination of β_0 and $\beta_{1/2}$.*

Remark that $\Lambda^n\ \beta_p$ does not converge as $n\rightarrow\infty$ unless $p=0,1/2,1$. But Theorem 4 in [3] says that if $0<p<1$

$$\lim 1/N\sum_{n=0}^{N-1}\Lambda^n\beta_p=\beta_{1/2}.$$

It is natural to ask if there are any other Λ-invariant measures outside Conv (M) [1]. The answer is "Yes, there are infinitely many" [4]. Let us show this in more general setting.

Let $n\geqq 3$ be an odd integer. A configuration x_n in Ω is defined as follows;

$$x_n\ (i)=\begin{cases}0 & \text{if } i=0 \bmod n,\\ 1 & \text{otherwise.}\end{cases}$$

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