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Root lattices and pencils of varieties

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1. Introduction

In this paper a particular pencil of K3 surfaces is investigated. The Picard-Fuchs equation, solutions, and the monodromy group are found. The paper [21] gives the Picard-Fuchs equation for several families of elliptic curves. Similar methods are also used in [14] and [15] to obtain the Picard-Fuchs equation for the case of certain pencils of K3 surfaces. In these papers finding the Picard-Fuchs equation comes down to finding a recurrence relation for some sequence of combinatorially defined terms, e.g., in [5], recurrence relations are found for a_n in the following cases:

$$a_n = \sum_{k=0}^{n} \binom{n}{k}^2 \binom{n+k}{k}, \sum_{k=0}^{n} \binom{n}{k}^3, \quad \text{and} \quad \sum_{k=0}^{n} \binom{n}{k}^2 \binom{2k}{k}.$$

In these examples, the symmetry of the defining equations of the varieties leads to the determination of the Picard-Fuchs equations from the combinatorial data. This gives motivation to try and find further examples, by considering other pencils of varieties with a good degree of symmetry.

The pencil of K3 surfaces in [15] is acted on by the Weyl group of the root lattice $A_1 \times A_1 \times A_1$, and the pencil can be viewed as being constructed from this lattice. This construction is described in section 2, for a general root lattice.

In this paper, the A_3 case is investigated. The corresponding pencil of K3 surfaces is denoted by $\mathscr{X}_{A3}$.

To find the Picard-Fuchs equation for $\mathscr{X}_{A_3}$, one has to find a recurrence relation for

$$a_n = \sum_{p+q+r+s=n} \binom{n}{pqrs}^2,$$

where $\binom{n}{pqrs} = \dfrac{n!}{p!q!r!s!}$.

The following result is obtained (cf. 4.3, Proposition 7):

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