J. Math. Kyoto Univ. (JMKYAZ)
36-2 (1996) 331–358

On Chern numbers of homology planes of certain types

By

Toru SUGIE and Masayoshi MIYANISHI

1. Introduction

A nonsingular algebraic surface X defined over $\mathbf{C}$ is called a homology plane if homology groups $H_i(X; \mathbf{Z})$ vanish for all $i > 0$. We know how to construct a homology plane with Kodaira dimension $\kappa(X) \leq 1$ (cf. [2]). As for homology planes with $\kappa(X) = 2$, though plenty of examples of such homology planes have been constructed, we are still far from classifying them completely.

Since a homology plane X is an affine rational surface and X has a fiber space structure whose general fibers are isomorphic to $\mathbf{C}^{N*}$, where $\mathbf{C}^{N*}$ is the affine line minus N points, it seems natural to begin with a study in the case $N = 2$, that is, a homology plane with a $\mathbf{C}^{**}$-fibration. Note that the case $N = 1$ corresponds to $\kappa \leq 1$. In our previous paper [5], we treated this case $N = 2$ and classified homology planes with $\mathbf{C}^{**}$-fibrations. In [1], tom Dieck gave several examples in the case $N = 3$. In this context, the following problem seems interesting.

Problem 1. *Let X be a homology plane of general type. Define the number $F(X)$ by*

$$F(X) = \min \{N \mid \text{there exists a } \mathbf{C}^{N*}\text{-fibration on } X\} .$$

Is $F(X)$ then bounded or not? Namely, does there exist a constant A independent of X such that $F(X) \leq A$?

The Chern numbers and the Miyaoka-Yau inequality play an important role in the classification theory of projective surfaces. The inequality gives the first restriction to the existence area of surfaces in the (c_2, c_1^2)-plane and further precise research is made for the surfaces corresponding to values in this area. We would like to use Chern numbers in the study of homology planes. The Miyaoka-Yau inequality was extended to the open surfaces in [3, 4] and the inequality $c_1^2 \leq 3c_2$ holds also for open surfaces if c_1^2 and c_2 stand for logarithmic Chern numbers. We note that if X is an open surface, c_1^2 could be a rational number. (See below for the definition of c_1^2.) Since Betti numbers of a homology plane X are zero except for b_0, the Euler number c_2 of X equals one.

Communicated by Prof. K. Ueno, April 21, 1995