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Decomposition of the canonical representation of $W(1) \times \mathrm{End}[m]$ on $\Lambda(m)$

Delicated to Professor Takashi Hirai on his 60th birthday

By

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Introduction. There are several ways to obtain finite dimensional irreducible representations of the general linear group $GL(n, \mathbf{C})$, or equivalently of its Lie algebra $\mathfrak{gl}(n, \mathbf{C})$. Among them the method introduced by Schur and Weyl is classical. They considered the m-fold tensor product $V^{\otimes m}$ of the defining representation of $GL(V)$ $(V=\mathbf{C}^n)$, and the action of the permutation group $\mathfrak{S}_m$ on it. These actions are commutative each other, and $V^{\otimes m}$ decomposes multiplicity freely as a $GL(V) \times \mathfrak{S}_m$-module:

$$V^{\otimes m} = \sum_D \rho_D \otimes \sigma_D,$$

where D runs over Young diagrams of size m with depth at most n,and ρ_D (resp. σ_D) is an irreducible representation of $GL(n)$ (resp. $\mathfrak{S}_m$). Furthermore, through the decomposition, ρ_D determines σ_D and vice versa (e.g., see [1]). If m varies in the set of non-negative integers, each irreducible representation of $GL(n, \mathbf{C})$ appears in this decomposition up to multiplication by a suitable power of the determinant character.

The Cartan-type Lie algebras are $\mathbf{Z}$-graded, simple, infinite-dimensional Lie algebras, whose properties and representations have been discussed extensively. I.A.Kostrikin ([6]) proved that all the finite type graded representations are either representations of height 1 (in the sense of A. N. Rudakov [11]) or their conjugate except the algebra $W(1)$. In the case of $W(1)$, Kostrikin gave models of all the irreducible graded modules of finite type with one-dimensional homogeneous components. On the other hand, K. Nishiyama ([8]) considered Schur-Weyl duality for the natural representation of Cartan-type Lie algebra $W(n)$. In particular, he suggested to use End $[m]$ insted of $\mathfrak{S}_m$, which is a semigroup consisted of all the mappings from a finite set $[m]=\{1, 2, \cdots, m\}$ into itself.

For Cartan-type Lie *superalgebras*, which is a "superanalogue" of Cartan-type Lie algebras, we also want to have an analogue of the Schur-Weyl duality. As a first step, we consider one of Cartan-type Lie superalgebras: Lie superalgebra of all the superderivations on the Grassmann algebra $\Lambda(n)$, which is denoted by $W(n)$ (see, e.g., [4]). In [9] and [10], using the semigroup End $[m]$, the author and Nishiyama have determined the

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