J. Math. Kyoto Univ. (JMKYAZ)
37-4 (1998) 615-638

Asymptotics of the Infimum of the Spectrum of Schrödinger Operators with Magnetic Fields*

By

Naomasa UEKI

1. Introduction

Let D be a domain in $\mathbf{R}^d$ given a Riemannian metric and b be a 1-form on D. Let $L(b)$ be the self-adjoint operator corresponding to the closed extension of the form

$$q(b)(\varphi)=\frac{1}{2}\|(id+\operatorname{ext}(b))\varphi\|^2, \quad i=\sqrt{-1}, \tag{1.1}$$

for any $\varphi\in C_0^\infty(\mathring{D})$: $L(b)$ is the Schrödinger operator with a magnetic field db and the Dirichlet boundary condition. For the notation, see Section 2 below.

In this paper we give some lower estimates of the asymptotics of the infimum, inf spec $L(\xi b)$, of the spectrum of the operator $L(\xi b)$ as the real parameter ξ tends to infinity. We intend particularly to its application to the study of the asymptotics of the function

$$I(\xi):=E\left[\exp\left(-i\xi\int_0^t b(X(s,x))\circ dX(s,x)\right)\middle|X(t,x)=y\right], \tag{1.2}$$

as ξ tends to infinity, where $X(s,x)$ is the absorbing barrier Brownian motion on a domain D, x, y are fixed points in D, and $E[\cdot|\cdot]$ is the conditional expectation. This is called the stochastic oscillatory integral in Malliavin [11], Ikeda and Manabe [7] and so on. The connection between the operator $L(\xi b)$ and the function $I(\xi)$ is given by the Feynman-Kac-Itô formula (see (2.17) below). By this formula and our estimate of inf spec $L(\xi b)$, we obtain some upper estimate of the absolute value $|I(\xi)|$ of $I(\xi)$. Accordingly, we obtain some results on the existence and the regularity of the density of the conditional probability with respect to the Lebesgue measure

$$P\left(\int_0^t b(X(s,x))\circ dX(s,x)\in d\lambda\middle|X(t,x)=y\right)\Big/d\lambda. \tag{1.3}$$

Communicated by Prof. K. Ueno, April 15, 1996
*This work was supported by Grant-in-Aid for Scientific Research, The Ministry of Education, Science and Culture, Japan