J. Math. Kyoto Univ. (JMKYAZ)
41-3 (2001), 557–592

Weak solutions to the compressible Euler equation with an asymptotic γ-law

Dedicated to Professors Takaaki Nishida and Masayasu Mimura
on their sixtieth birthdays

By

Tetu Makino

1. Introduction

The one-dimensional motion of a perfect gas is governed by the compressible Euler equation

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) &= 0, \\
\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2 + P) &= 0,
\end{aligned} \tag{1.1}
$$

where unknowns are the density ρ and the velocity u, while the pressure P is supposed to be a given function of ρ. We study the Cauchy problem to the equation under the initial condition

$$
\rho|_{t=0} = \rho_0(x), \qquad u|_{t=0} = u_0(x). \tag{1.2}
$$

The equation is a prototype of the conservation law

$$
U_t + f(U)_x = 0, \tag{1.3}
$$

in which

$$
U = (\rho, m)^T = (\rho, \rho u)^T, \qquad f(U) = \left(m, \frac{m^2}{\rho} + P \right)^T.
$$

A bounded measurable function $U(t,x)$ is a weak solution if

$$
\int_0^\infty \int (U\Phi_t + f(U)\Phi_x) dx dt + \int \Phi(0,x) U_0(x) dx = 0
$$

Communicated by Prof. T. Nishida, August 25, 2000