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An approach to the biharmonic pseudo process by a random walk

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1. Introduction

We consider the partial differential equation

$$\frac{\partial u}{\partial t} = -\frac{1}{8}\frac{\partial^4 u}{\partial x^4}. \tag{1.1}$$

Here $\Delta^2 = \partial^4/\partial x^4$ is called the biharmonic operator. The fundamental solution of this equation is

$$q(t,x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\lambda e^{-ix\lambda-\frac{1}{8}\lambda^4 t}. \tag{1.2}$$

As shown by Hochberg [H], this function is not positive valued. Hence the usual probabilistic method is not applicable. However, there are several probabilistic approaches to this equation.

Krylov [K], later Hochberg [H] and Nishioka [N1] considered a signed finitely additive measure on a path space, which is essentially the limit of a Markov chain with the signed distribution $q(t,x)$. Krylov obtained a path continuity of the *biharmonic pseudo process*. Note that there does not exist the σ-additive measure on a path space realizing a stochastic process related to the biharmonic operator. We can consider the biharmonic pseudo process only through a limit procedure from a finite additive measure. Our aim of this paper is to propose a new limit procedure for the biharmonic pseudo process.

Nishioka considered the first hitting time to $(-\infty,0)$ and obtained the joint distribution of the first hitting time and the first hitting place. His remarkable result is that there appears the differentiation at 0 for the distribution of the first hitting place (see Section 6). However, since the jump distribution $q(t,x)$ is spread on the entire of $\mathbf{R}$, it is difficult to understand this fact intuitively.

In this paper, we define a random walk, which has the only finite jumps with signed measure. And we will show Nishioka's result by a scaling limit. We can get short and elementary proofs for his result. And we can understand that

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